

Entanglement dynamics in superconducting qubits affected by local bistable impurities

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Abstract. We study the entanglement dynamics for two independent superconducting qubits each affected by a bistable impurity generating random telegraph noise (RTN) at pure dephasing. The relevant parameter is the ratio g between qubit-RTN coupling strength and RTN switching rate, that captures the physics of the crossover between Markovian and non-Markovian features of the dynamics. For identical qubit-RTN subsystems, a threshold value g_{th} of the crossover parameter separates exponential decay and onset of revivals; different qualitative behaviors also show up by changing the initial conditions of the RTN. We moreover show that, for different qubit-RTN subsystems, when both qubits are very strongly coupled to the RTN an increase in entanglement revival amplitude may occur during the dynamics.

PACS numbers: 03.67.Mn, 03.65.Yz, 03.67.-a

1. Introduction

Entanglement dynamics has been intensively studied for bipartite quantum systems interacting with quantum environments (independent or common). It presents, depending on the Markovian or non-Markovian nature of the environments, phenomena like entanglement sudden death (ESD) [?], revivals [?, ?, ?] or trapping [?, ?]. Entanglement dynamics has been also analyzed in some solid-state systems, like superconducting qubits or quantum dots, which are promising candidates for realizing quantum information processing [?, ?, ?, ?]. A considerable progress has been made in the last decade towards the implementation of a controlled solid-state quantum computer. In particular, superconducting high-fidelity single qubit gates with coherence times of $\sim 1\mu\text{s}$ are currently available [?, ?]. High-fidelity Bell states of superconducting qubits have been also prepared [?, ?]. Superconducting nanodevices are usually affected by broadband noise, with typical power spectra displaying a $1/f$ low-frequency behavior followed by a white or ohmic flank [?, ?, ?, ?]. Dynamics of entanglement in these devices has been recently analyzed distinguishing the effect of adiabatic and quantum noise [?]. The disentanglement process has been also studied by the quasi-Hamiltonian method when two independent superconducting qubits are locally affected by random telegraph noise (RTN) [?].

In this paper we consider a model, relevant to solid-state system [?], consisting of two noninteracting qubits each affected by a single bistable impurity inducing RTN at the pure dephasing working point. In addition, this model can be exactly solvable in analytic form and captures the physics of the crossover between a Markovian and non-Markovian environment under quite general conditions. The relevant parameter separating the two regimes is the ratio between qubit-RTN coupling constant and switching rate of the impurity. We analyze the entanglement dynamics both for identical and different conditions of the two local qubit-RTN. In Sec. 2 we introduce the relevant single qubit-RTN model, while in Sec. 3 we describe the entanglement dynamics under various noise initial conditions. In Sec. 4, we present our conclusive remarks.

2. Model: a qubit under pure dephasing RTN

Our system consists of a couple of independent superconducting qubits, A and B , each affected by a bistable impurity generating RTN at pure dephasing. The total Hamiltonian is $H_{\text{tot}} = H_A + H_B$, where for each qubit the Hamiltonian at pure-dephasing is ($\hbar = 1$) [?] $H = -(\Omega/2)\sigma_z - (v/2)\xi(t)\sigma_z$, where $\xi(t)$ is a stochastic process producing RTN switching at a rate γ between ± 1 and v is the qubit-RTN coupling constant. The power spectrum of the unperturbed equilibrium fluctuations of $\xi(t)$ is $s(\omega) = v^2\gamma/[2(\gamma^2 + \omega^2)]$. The relevant parameter is the ratio $g = v/\gamma$ that permits to analyze the crossover between a Markovian environment for weakly coupled impurities ($g < 1$) and a non-Markovian environment for strong coupled impurities ($g > 1$) [?]. The exact evolution of single-qubit coherence $q(t) \equiv \rho_{01}(t)/\rho_{01}(0)$ has been found in

Ref. [?] and it is given by

$$q(t) = e^{-i(\Omega+v/2)t} [Ae^{-\frac{\gamma(1-\alpha)t}{2}} + (1-A)e^{-\frac{\gamma(1+\alpha)t}{2}}], \quad (1)$$

where $A = \frac{1}{2\alpha}(1 + \alpha - ig\delta p_0)$ and $\alpha = \sqrt{1 - g^2}$. δp_0 is a degree of freedom depending on the initial conditions of the RTN and is not present in previous analysis of entanglement dynamics under RTN [?]. In particular, δp_0 can take the value $\delta p_0 = 0$ (corresponding to the value of thermodynamical equilibrium) or the values $\delta p_0 = \pm 1$. The form of Eq. (1) clearly shows the different roles of weakly and strongly coupled impurities in the decoherence process. Dephasing comes from the sum of two exponential terms. If $g \ll 1$ only the first of these terms is important and the corresponding rate is $\approx v^2/(4\gamma)$, coinciding with the golden rule; if $g \gg 1$ the two terms are of the same order and the decay rate is $\sim \gamma$ [?].

3. Dynamics of entanglement

The two-qubit density matrix elements are evaluated in the computational basis $\mathcal{B} = \{|0\rangle \equiv |00\rangle, |1\rangle \equiv |01\rangle, |2\rangle \equiv |10\rangle, |3\rangle \equiv |11\rangle\}$, where $H_i|0_i\rangle = -\frac{\Omega_i}{2}|0_i\rangle$, $H_i|1_i\rangle = \frac{\Omega_i}{2}|1_i\rangle$ ($i = A, B$). We consider as initial states the extended Werner-like (EWL) states expressed by the density matrices [?]

$$\hat{\rho}_1 = r|1_a\rangle\langle 1_a| + \frac{1-r}{4}\mathbb{1}_4, \quad \hat{\rho}_2 = r|2_a\rangle\langle 2_a| + \frac{1-r}{4}\mathbb{1}_4, \quad (2)$$

whose pure parts are the one-excitation and two-excitation Bell-like states $|1_a\rangle = a|01\rangle + b|10\rangle$, $|2_a\rangle = a|00\rangle + b|11\rangle$, where the subscript a identifies the initial degree of entanglement of the pure part and $|a|^2 + |b|^2 = 1$. The density matrix of EWL states is non-vanishing only along the diagonal and anti-diagonal (X form) [?] and this structure is maintained at $t > 0$ in the system we are considering (pure dephasing). Using the concurrence [?] C to quantify entanglement, the initial entanglement is equal for both the EWL states of Eq. (2) and reads $C_{\rho_1}(0) = C_{\rho_2}(0) = 2\max\{0, (|ab| + 1/4)r - 1/4\}$. Initial states are thus entangled for $r > r^* = (1 + 4|ab|)^{-1}$. Moreover, the purity $P = \text{Tr}(\rho^2)$ of EWL states is $P = (1 + 3r^2)/4$. Entangled states with purity ≈ 0.87 and fidelity to ideal Bell states ≈ 0.90 have been experimentally generated [?]: these states may be approximately described as EWL states with $r_{\text{exp}} \approx 0.91$.

In order to obtain the concurrence at time t , we need the evolved two-qubit density matrix which can be evaluated by the knowledge of the single-qubit density matrix evolution, according to a standard procedure [?]. The initial EWL states, during the pure-dephasing evolution, maintain the diagonal elements unchanged and the anti-diagonal elements depend on product of single-qubit coherences. The concurrences at time t for the two initial states of Eq. (2) are given by, respectively, $C_{\rho_1}(t) = 2\max\{0, K_1(t)\}$ and $C_{\rho_2}(t) = 2\max\{0, K_2(t)\}$ where $K_1(t) = |\rho_{12}(t)| - \sqrt{\rho_{00}(t)\rho_{33}(t)}$, $K_2(t) = |\rho_{03}(t)| - \sqrt{\rho_{11}(t)\rho_{22}(t)}$. In our case we have $K_1(t) = K_2(t) = K(t)$, so that the two concurrences are equal for both initial states $C_{\rho_1}(t) = C_{\rho_2}(t) = C(t)$. In the following, we consider identical qubits ($\Omega_A = \Omega_B = \Omega$) and we distinguish the cases of equal RTNs $g_A = g_B = g$ and of different RTNs $g_A \neq g_B$ for the two subsystems.

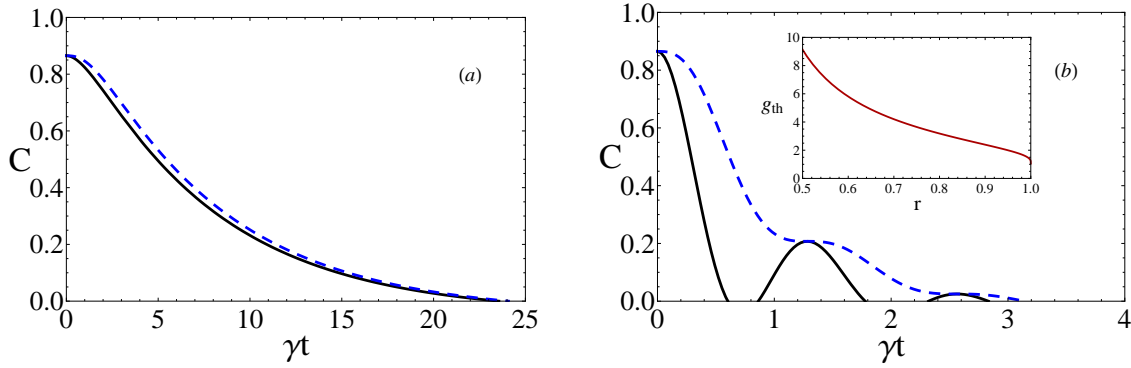


Figure 1. Concurrences versus the dimensionless time γt with $r = r_{\text{exp}} = 0.91$ and $a = b = 1/\sqrt{2}$ for $g = 0.5$ (panel (a)) and $g = 5$ (panel (b)). Solid black lines are for $\delta p_0 = 0$ while dashed blue lines for $\delta p_0 = \pm 1$. The inset of panel (b) displays g_{th} versus r for $a = b = 1/\sqrt{2}$.

3.1. Identical RTNs

Consider the case $g_A = g_B = g$ and equal initial conditions. We obtain $C(t) = \max\{0, 2K(t)\}$ with

$$K(t) = r|a|\sqrt{1 - |a|^2}|q(t)|^2 - (1 - r)/4, \quad (3)$$

where $q(t)$ is the single-qubit coherence of Eq. (1). Now we investigate how the concurrence depends on the different initial conditions of the system and state parameters. Equation (3) clearly shows the role of the purity parameter r on concurrence evolution: for pure states, $r = 1$, there cannot be an entanglement sudden death (ESD) and the entanglement goes asymptotically to zero similarly to single-qubit coherence; on the other hand, any value of r inside the interval $r^* < r < 1$ determines a threshold value for the first term of Eq. (3) to be overcome in order to have nonzero entanglement (of course, $0 < |a| < 1$). The dynamics of entanglement is shown in Fig. 1, when $g = 0.5, 5$ and $\delta p_0 = 0, \pm 1$, for initial states with $r = r_{\text{exp}} = 0.91$ and $a = b = 1/\sqrt{2}$ whose initial concurrence is $C(0) = 0.865$. Similarly to other works [?, ?], for weak coupling ($g < 1$, Markovian environment) there is a simple (exponential) decay while for strong coupling ($g > 1$, non-Markovian environment) there are damped revivals after dark periods of entanglement (see Fig. 1). It is however worth to note that, while for a single qubit the crossover between Markovian (exponential) and non-Markovian (oscillating) decay is identified by $g = 1$, for the two-qubit system, depending on initial state parameters, a threshold value $g_{\text{th}} > 1$ exists separating exponential decay and the onset of revivals. This g_{th} can be found analytically [?] and its dependence on r (for $a = b = 1/\sqrt{2}$) is displayed in Fig. 1(b) (for $r = 1$ it is $g_{\text{th}} = 1$). When $r = 1$ the entanglement goes to zero asymptotically (oscillating and vanishing at given times for $g > g_{\text{th}}$), analogously with the results already found for the case of adiabatic (low-frequency) noise due to collective impurities generating $1/f$ -noise [?]. Differently, when $r < 1$ there is always ESD for $g \leq g_{\text{th}}$ and a “final death” (after revivals) for $g > g_{\text{th}}$,

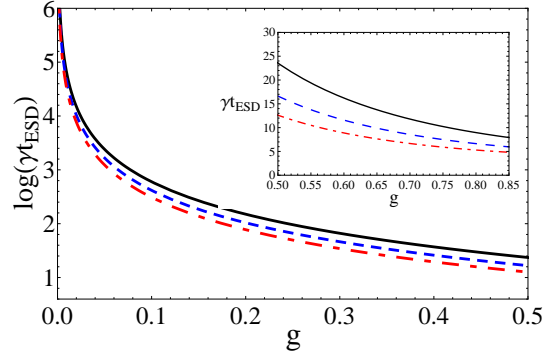


Figure 2. ESD times t_{ESD} (scaled with γ) versus g , with $a = b = 1/\sqrt{2}$ and $\delta p_0 = 0$, for $r_{\text{exp}} = 0.91$ (black solid line), $r = 0.8$ (blue dashed line) and $r = 0.7$ (red dot-dashed line). The inset shows the range $g > 0.5$.

where for final death we mean the definitive disappearance of entanglement. Different RTN initial conditions (values of δp_0) qualitatively affects the entanglement dynamics for $g > g_{\text{th}}$ (strong memory effects), while leave it practically unchanged for $g \leq g_{\text{th}}$ (weak memory effects), as shown in Fig. 1. In particular, when $g > g_{\text{th}}$, the concurrence for $\delta p_0 = \pm 1$ is always larger than that for $\delta p_0 = 0$ with small beats touching the peaks of the revivals appearing in the corresponding curve for $\delta p_0 = 0$; the final death time is also longer than the previous one (see Fig. 1).

In the limit of small g (Markovian noise) and for $\delta p_0 = 0$, only the first term is important in Eq. (1) and, from Eq. (3), we may estimate the ESD times (giving however good match with plots up to $g = 0.9$)

$$\gamma t_{\text{ESD}} = -\frac{2}{1 - \sqrt{1 - g^2}} \ln \left(\frac{\sqrt{\frac{(1-g^2)(1-r)}{r|a|\sqrt{1-|a|^2}}}}{1 + \sqrt{1 - g^2}} \right). \quad (4)$$

These ESD times are plotted in Fig. 2 as a function of g for three different values of r . They decrease as g increases, with a reduction of about an order of magnitude going from $g = 0.1$ to $g = 0.4$.

3.2. Different RTNs

We now consider the more realistic case of qubits affected by different RTNs $g_A \neq g_B$, with $\delta p_0 = 0$ for both. The function $K(t)$ is now given by Eq. (3) replacing $|q(t)|^2$ with $|q_A(t)q_B(t)|$, where $q_i(t)$ ($i = A, B$) is the coherence of qubit i given in Eq. (1). The entanglement dynamics for different values of g_A, g_B is displayed in Fig. 3. We assume the rate γ fixed for both impurities and different values of v_A, v_B . When a qubit, for instance qubit B , is not affected by RTN ($g_B = 0$) we find a qualitative behavior analogous to that observed for identical RTNs, with a threshold value $g_{A\text{th}} > 1$ after which revivals occur. On the other hand, when one of the two qubits is affected by

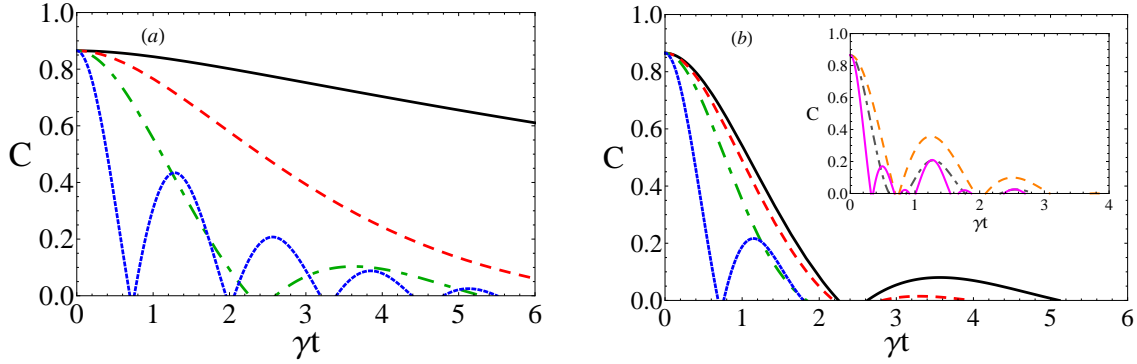


Figure 3. Concurrences versus γt for $g_B = 0$ (panel (a)) and $g_B = 2$ (panel (b)) and values of g_A equal to: 0.5 (solid black line), 1.1 (red dashed line), 2 (green dot-dashed line), 5 (blue dotted line). In the inset of panel (b) we set $g_B = 5$ with values of g_A equal to: 1.1 (orange dashed line); 5 (gray dot-dashed line); 10 (magenta solid line). Initial conditions: $r = r_{\text{exp}} = 0.91$, $a = b = 1/\sqrt{2}$, $\delta p_0 = 0$.

a strong non-Markovian RTN (for instance, $g_B = 2$) when g_A increases the revivals tend to disappear after a certain $g_A > 1$ (indeed, when $g_A = g_B = 2$ the value of g_{th} is larger than 2, as seen in previous section) and then reappear for larger values of g_A with final death times shorter and shorter. By further increasing the fixed value of g_B , other new qualitative behaviors appear when g_A increases in the strong coupling domain. Inset of Fig. 3(b) shows the presence of entanglement revivals whose amplitude increases with respect to previous one for the values $g_A = 10, g_B = 5$. Similar behaviors occur for initial pure states ($r = 1$) but without dark periods. These behaviors are due to the different contributions of the two single-qubit dynamics.

4. Conclusions

In this work we have analyzed the entanglement dynamics in a system of two initially entangled independent superconducting qubits (A and B) each affected by impurity-induced random telegraph noise at pure dephasing. A crucial role in determining the behavior of entanglement dynamics is played by the ratio between qubit-RTN coupling v and switching rate γ , $g = v/\gamma$. For identical RTNs we have found that, in spite of the fact that for the single-qubit dynamics the crossover between Markovian and non-Markovian behavior is identified by $g = 1$, for an initially entangled two-qubit system this threshold is in general shifted to a value $g_{\text{th}} > 1$ depending on the initial state (for initial pure states $g_{\text{th}} = 1$). In the case when the RTN has the initial value $\delta p_0 = 0$ (thermodynamical equilibrium), we have retrieved the known behaviors of exponential (Markovian) decay for $g \leq g_{\text{th}}$ and of oscillating (non-Markovian) decay with revivals for $g > g_{\text{th}}$. We have then found new qualitative behaviors, in the non-Markovian regime ($g > g_{\text{th}}$, relevant memory effects), by changing the initial value of RTN, δp_0 . For example, if $\delta p_0 = \pm 1$, the entanglement never vanishes at intermediate times and no revivals occur, with a final death time longer than that for $\delta p_0 = 0$. We have

finally considered the case when the two local qubit-RTN conditions differ, in particular $g_A \neq g_B$. Entanglement dynamics may behave quite differently than that for identical subsystems. In particular, when both g_A, g_B are in the strong non-Markovian domain a new feature in the entanglement dynamics shows up, that is there can be entanglement revivals whose amplitude increases with respect to previous one.

The simple model, relevant to solid-state systems, analytically studied here displays the richness of behaviors of entanglement dynamics and its crucial dependence both on system-environment parameters and on initial conditions.

Acknowledgments

Partially supported by EU through Grant No. PITN-GA-2009-234970.

References